

# Three-port S-parameters ease GaAs FET designing

A simple technique saves design time by simplifying conversions from two- to three-port S-parameters.

THREE-port S-parameters, introduced quite some time ago, have not been popular due to the complexity of the analysis.<sup>1</sup> Hence, the GaAs FET, a three-port device, has usually been characterized by two-port S-parameters, with one of its ports grounded. Now, because desktop computers and CAD programs make easy work of complex analysis tasks, three-port S-parameters are a practical tool for characterizing GaAs FETs. The use of three-port parameters eliminates unnecessary conversion to and from Z and Y parameters to analyze series and parallel feedback effects.

The three configurations of a GaAs FET circuit are shown in Fig. 1. Each has its advantage in various applications. For example, the common-source setup is used most often in amplifiers, and the common-gate configuration for wideband oscillators. The common-drain circuit frequently finds service in medium-power oscillators. Using the technique explained here, the S-parameters of one configuration can easily be converted into the S-parameters of another.

The sums of the rows and columns of the three-port indefinite S-matrix of a

GaAs FET are each equal to one. That relationship, along with three-port S-parameters, helps to determine and eventually to correct systematic errors in measurement or analysis, and is a useful tool for circuit analysis.<sup>2</sup> The three-port indefinite S-matrix is first found from the given two-port S-parameters, and then it is converted to the two unknown two-port-configuration S-parameters.

The transistor in Fig. 2 is a three-port device. The S-matrix of the incident and reflected waves is given by

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad (1)$$

where  $a$  represents the reflective- and  $b$ , the incident-wave parameters. The terminals designated 1, 2, and 3 represent the gate, drain, and source, respectively. This indefinite three-port

S-matrix satisfies the following conditions:

$$\sum_{j=1}^3 S_{ij} = 1 \quad (2)$$

$$\sum_{i=1}^3 S_{ij} = 1 \quad (3)$$

where  $i$  and  $j$  represent any number between 1 and 3.

The relationship between Eqs. 2 and 3 represents six equations in terms of the nine three-port S-parameters that must be determined. Other equations needed to find all nine three-port S-parameters are found as follows:

From equation 1 comes

$$b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 \quad (4)$$

$$b_2 = S_{21}a_1 + S_{22}a_2 + S_{23}a_3 \quad (5)$$

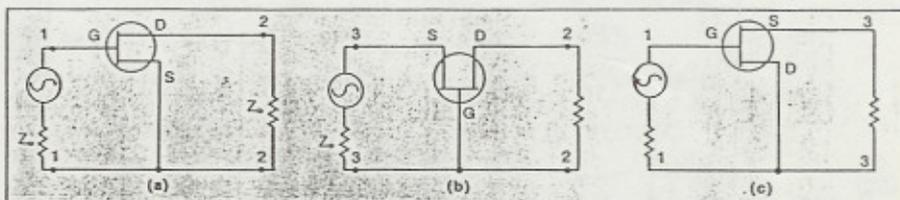
$$b_3 = S_{31}a_1 + S_{32}a_2 + S_{33}a_3 \quad (6)$$

Thus, for example, if Port 3 of the transistor in Fig. 2 is connected to ground, making it a common-source configuration, then

$$\Gamma_3 = \frac{a_3}{b_3} = -1 \quad (7)$$

where  $\Gamma_3$  is the reflection coefficient of Port 3.

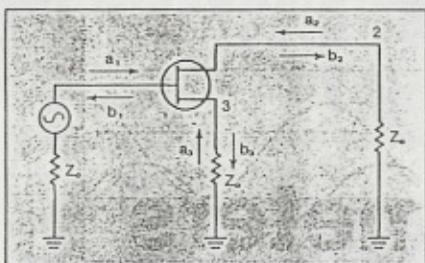
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1. A three-port GaAs FET is characterized by its two-port S-parameters. The two-port connections are common source (a), common gate (b), and common drain (c).

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## S-parameters (continued from p. 81)



2. The configuration of a GaAs FET as a three-port device aids in the analysis of series and parallel feedback effects.

Using Eq. 7 in Eqs. 4 and 6, and eliminating  $a_3$  and  $b_3$ , yields

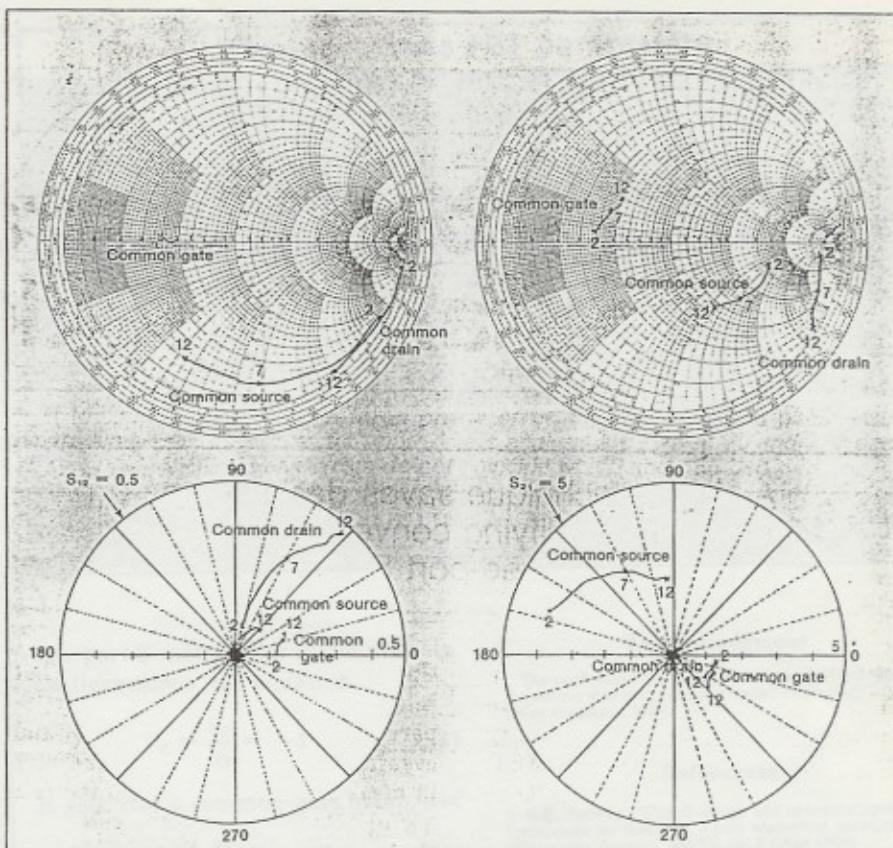
(See Eq. 8 below)

or

$$[b] = [S^T][a] \quad (9)$$

where  $[S^T]$  represents the known reduced two-port S-matrix of the FET with its third terminal connected to ground, and Eq. 8 represents four equations, which combined with the six equations from Eqs. 2 and 3, stand for a total of 10 equations that must be solved to find all nine three-port S-parameters. The final relationships that convert a given two-port S-matrix into an indefinite three-port S-matrix are Eqs. 10 to 18. They represent the necessary relations to convert a given two-port S-matrix directly into an indefinite three-port S-matrix without the need for converting to and from Z and Y parameters.<sup>3,4</sup> The simple relationships are as follows:

$$S_{33} = \frac{\sum_{i=1,2} S_{ij}^T}{4 - \sum_{i=1,2} S_{ij}^T} \quad (10)$$



3. The design technique yields an S-parameter comparison of three two-port configurations for the NEC 70000 GaAs FET operating from 2 to 12 GHz.

$$S_{32} = \frac{1 + S_{33}}{2} (1 - S_{12}^T - S_{22}^T) \quad (11)$$

$$S_{21} = 1 - S_{22} - S_{23} \quad (17)$$

$$S_{23} = \frac{1 + S_{33}}{2} (1 - S_{21}^T - S_{22}^T) \quad (12)$$

$$S_{11} = 1 - S_{21} - S_{31} \quad (18)$$

$$S_{22} = S_{22}^T + \frac{S_{23}S_{32}}{1 + S_{33}} \quad (13)$$

$$S_{13} = 1 - S_{23} - S_{33} \quad (14)$$

$$S_{31} = 1 - S_{33} - S_{32} \quad (15)$$

$$S_{12} = 1 - S_{22} - S_{32} \quad (16)$$

Eqs. 4 through 6 convert three-port S-parameters to two-port S-parameters. For a common-source configuration,  $\Gamma_3 = a_3/b_3 = -1$  is substituted in the equations to arrive at Eq. 8, which represents the common-source (concluded on p. 84)

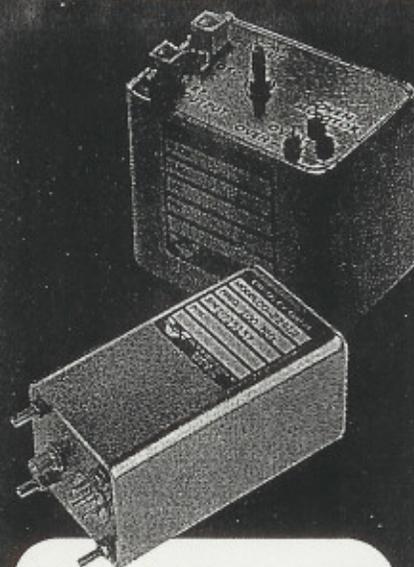
$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} - \frac{S_{13}S_{31}}{1+S_{33}} & S_{12} - \frac{S_{13}S_{32}}{1+S_{33}} \\ S_{21} - \frac{S_{31}S_{23}}{1+S_{33}} & S_{22} - \frac{S_{32}S_{23}}{1+S_{33}} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} S_{11}^T & S_{13}^T \\ S_{31}^T & S_{33}^T \end{bmatrix} = \begin{bmatrix} S_{11} - \frac{S_{12}S_{21}}{1+S_{22}} & S_{13} - \frac{S_{12}S_{23}}{1+S_{22}} \\ S_{31} - \frac{S_{32}S_{21}}{1+S_{22}} & S_{33} - \frac{S_{32}S_{23}}{1+S_{22}} \end{bmatrix} \quad (21)$$

$$\begin{bmatrix} S_{33}^T & S_{32}^T \\ S_{23}^T & S_{22}^T \end{bmatrix} = \begin{bmatrix} S_{33} - \frac{S_{13}S_{31}}{1+S_{11}} & S_{32} - \frac{S_{12}S_{31}}{1+S_{11}} \\ S_{23} - \frac{S_{13}S_{21}}{1+S_{11}} & S_{22} - \frac{S_{12}S_{21}}{1+S_{11}} \end{bmatrix} \quad (22)$$

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## DESIGN FEATURE

### S-parameters (continued from p. 82)

#### Two-port GaAs FET parameters

Frequency (GHz)	K			G <sub>T</sub> (dB)			MSG (dB)			MUG (dB)		
	2	7	12	2	7	12	2	7	12	2	7	12
Common source	0.23	0.5	0.93	11.0	8.6	6.1	19.5	14.7	13.0	23.6	14.8	10.1
Common gate	0.94	0.6	0.4	2.5	2.6	2.8	10.8	9.8	8.8	10.3	13.2	20.1
Common drain	0.11	0.13	0.20	2.3	2.0	1.5	11.6	5.8	3.6	20.1	19.6	14.3

where:

$$K = \text{the stability factor} = \frac{(1 + \Delta^2 - |S_{11}|^2 - |S_{22}|^2)}{2|S_{12}S_{21}|}$$

with  $\Delta = S_{11}S_{22} - S_{12}S_{21}$

$$G_T = \text{the transducer power gain} = 10 \log |S_{21}|^2$$

$$\text{MSG} = \text{the maximum stable power gain} = 10 \log \left( \frac{|S_{21}|^2}{|S_{12}|^2} \right); \text{ and}$$

$$\text{MUG} = \text{the maximum unilateral power gain} = 10 \log \frac{|S_{21}|^2}{(1 - |S_{11}|^2)(1 - |S_{22}|^2)}$$

two-port S-matrix. For common-drain configurations, the equation

$$\Gamma_2 = \frac{a_2}{b_2} = -1 \quad (19)$$

is used; for a common-gate setup, use

$$\Gamma_1 = \frac{a_1}{b_1} = -1 \quad (20)$$

For a common-drain configuration with a gate input and a source output (Fig. 1c), the equation needed to relate the two-port S-parameters to three-port S-parameters is

(See Eq. 21 at bottom of p. 82.)

For a common-gate circuit with a source input and a drain output (Fig. 1b), the equation is

(See Eq. 22 on bottom of p. 82.)

Both equations can be generalized as

$$S_{ji}^T = S_{ji} - \frac{S_{ki}S_{jk}}{1 + S_{kk}} \quad (23)$$

where k represents 1, 2, 3, which are the common gate, drain, and source configurations, respectively.

To illustrate the technique, two-port S-parameters supplied by the manufacturer of the NEC 70000 FET for common-source operation have been converted to common-drain and common-gate two-port S-parameters using the above equations. Figure 3 shows the S-parameters of the three two-port configurations for operation from 2 to 12 GHz. The table presents the important parameters of all three two-port configurations. ••

#### Acknowledgment

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#### References

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