Quality Factor Measurement of the Twoport Resonant Circuits Using Network Analyzer

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Abstract:

This paper describes the measurement of unloaded (Qu) and loaded (QL) quality factors of all the four configurations of the two port resonant circuits using S-parameters. Scattering matrix representation of the resonators is described and the necessary relations are derived to draw the Qu and QL loci on the network analyzer polar display in S_{11} and S_{21} plane of the resonator.

Introduction:

Resonant circuits form an important part of the microwaves and R.F. circuits. In addition to their use as circuit elements in oscillators, filters, frequency discriminators, amplifiers, mixers, etc; these are also used as tools for the measurements of frequency, oscillators noise, Radar ringtime etc. Their proper characterization is necessary for their effective use. In the vicinity of the resonance each resonator may be defined by an equivalent lumped R,L,C resonant circuit. The basic parameters of a resonant circuit are the resonant frequency fo, the unloaded Q factor Qu and the coupling factor . Ginzton (1) introduced the plotting of locii of various quality factors in the impedance plane of a single port resonant circuit. Kajfez (2) recently extended this work to use network analyzer techniques. A two port resonant circuit consisting of R,L,C can be represented in four different series/ parallel configurations as shown in figure 1. The quality factor measurement of one of the configurations (fig 1d) has already been described elsewhere [3] while discussing a dielectric resonator coupled to a microstrip line.

This paper presents a method, for the measurement of loaded (QL) and unloaded (Qu) quality factors as well as the coupling coefficient of all the four configurations of the two port - resonant RLC, series/parallel circuits (fig. 1). Use is made of the 2 port S-parameters directly measurable with the network analyzer to draw the loci for the Qu and QL on the polar display in the impedance (S_{11}) as well as the transmittance (S_{21}) plane of the resonator. The approach presented, in addition to help increase the measurement speed, clearly brings out the differences in the impedance (S_{11}) and transmittance (S21) loci of the different resonant configurations. The method can also be used to identify the correct circuit configuration from the known S-parameters around the resonant frequency.

S-parameters Characterization of the two port resonant circuits:

Microwaves and R.F. resonators can be described in various ways. Choice of the description form is decided by the particular application. The most universal is the rigorous "field description" based on the Maxwell equations. These equations are solved for different boundary conditions to determine the resonant frequency and quality factors of the resonators. The well known "circuits description" makes use of the equivalent circuits with lumped constants. The resonator parameters like resonant frequency fo, quality factors Qu and QL are determined from the circuit values using simple relations.

"Scattering-matrix description" of resonators is relatively little known. A general method was recently presented [4] to describe microwave resonators using S-parameters expressed in terms of frequency, quality factors and coupling coefficients. Either field or circuit

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description can be used to determine these parameters. The use of S-matrix simplifies the resonator parameter measurement and analysis of complex circuits involving resonators. The network analyzer which directly measures the s-parameters, makes the measurement of scattering matrix defined resonator parameters much simpler.

At the resonant frequency the S-matrix of all the four symmetrical configurations shown in figure 1 can be given by:

$$\begin{bmatrix} S_{Ro} \end{bmatrix} = \begin{bmatrix} S_{11o} & S_{12o} \\ S_{21o} & S_{22o} \end{bmatrix} = \begin{bmatrix} A & B \\ B & A \end{bmatrix}$$

A and B, the reflection and transmission coefficients of the resonant circuit at the resonant frequency are functions of the coupling coefficient β which also relates the various quality factors by the well know relation:

$$Qu = QL\ (1+\beta) = \beta\ Qex$$

The following table shows the relation between i) Qu, β and circuit elements ii) S-parameters A,B, and β, for all the configurations shown in figure 1:

Configuration	Qu	β	A	B
PRXP	R/WL	2R/Zo	$-(1+\beta)^{-1}$	$\beta(1-\beta)^{-1}$
PRXS	R/WL	R/2Zo	$\beta(1+\beta)^{-1}$	$(1-\beta)^{-1}$
SRXS	WL/R	2Zo/R	$(1+\beta)^{-1}$	$\beta(1-\beta)^{-1}$
SRXP	WL/R	Zo/2R	$-\beta(1+\beta)^{-1}$	$(1-\beta)^{-1}$

<u>Loci of Unloaded and Loaded Quality Factors:</u>

The procedure is presented below to draw the loci of the points on the reflectance (S_{11}) and transmittance (S_{21}) planes, for the frequency deviation corresponding to Qu and QL for different configurations. It may be noted that the relation between the reflection coefficient and transmission coefficient for the shunt impedance configurations PRXP and SRXP is given by $S_{21} = 1 + S_{11}$ and for series impedance configuration PRXS and SRXS, the relation is $S_{21} = 1 - S_{11}$. The input impedance/admittance for each configuration is first determined in terms of β and Qu. Loci of Qu and QL are then determined in S_{11} and S_{21} plane as shown:

i) configuration PRXP (fig.1a):

The coupling coefficient in terms of S-parameters is given by:

$$\beta = \frac{2R}{Zo} = \frac{S_{21o}}{1-S_{21o}} = \frac{1+S_{11o}}{-S_{11o}}$$

The input admittance y_{in} is given by:

$$y_{in} = 1 + g\ (1+2jQu\delta) \qquad \delta = (f-fo)/fo$$

$$= 1 + \frac{2(1+2jQu\delta)}{\beta}$$

$$= 1 + \frac{2\ (\ 1+2jQL\ (1+\beta)\delta)}{\beta}$$

The normalized frequency deviations corresponding to Qu and QL are given by:

$$\delta_u = \pm\frac{1}{2Q_u} \ \&\ \delta_L = \pm\frac{1}{2Q_L}$$

The admittance locus of Qu, for example, can now be given by:

$$Y_{inu} = 1 + \frac{2\ (\ 1\pm j\ )}{\beta}$$

or in terms of S_{21o} :

$$Y_{inu} = 1 + \frac{2\ (\ 1\pm j)\ (1-S_{21o})}{S_{21o}}$$

The corresponding reflection coefficient S_{11u} and transmission coefficient S_{21u} can be determined from:

$$S_{11u} = (1-y_{in})/(1+y_{in})\ \&\ S_{21u} = 1+S_{11u}$$

Using the above equations the final relations for the Qu loci on the S_{11} and S_{21} plane are given by:

$$S_{11u} = \frac{\sqrt{2}\ S_{11o}}{\sqrt{1+S_{11o}^2}} \text{Arg } \tan^{-1}\left(\frac{1-S_{11o}}{1+S_{11o}}\right)$$

$$S_{21u} = \frac{S_{21o}}{\sqrt{S_{21o}^2 - 2\ S_{21o} + 2}} \text{Arg } \tan^{-1}\ (S_{21o}-1)$$

Similarly the relations for the loci of QL in the S_{11} and S_{21} plane are given by:

$$S_{11L} = \sqrt{\frac{1+S_{11o}^2}{2}} \text{Arg } \tan^{-1}\left(\frac{1+S_{11o}}{1-S_{11o}}\right)$$

$$S_{21L} = \frac{S_{21o}}{\sqrt{2}} \text{Arg } 45^{o}$$

The above relations have been used to draw the Qu and QL loci on the S_{11} and S_{21} plane in figure 2. S_{11} and S_{21} curves for $\beta = 1$ have also been shown in figure 2. As and example $Qu= fo/(f1-f2)$ and $QL= fo/(f3-f4)$ can be directly determined from the polar display of the network analyzer. The direction of the increasing frequency is also shown on the figure.

ii) Configuration SRXS (fig. 1b):

Coupling coefficient β in terms of S-parameters is given by:

$$\beta = \frac{2Zo}{R} = \frac{1-S_{11o}}{S_{11o}} = \frac{S_{21o}}{1-S_{21o}}$$

$$z_{in} = 1 + 2\beta(1+j2Qu\delta)$$

&

$$S_{11} = (z_{in}-1)/(z_{in}+1)$$

using the similar approach as in PRXP case the final relations for the loci of Qu and QL in the S_{11} and S_{21} plane are given by:

$$S_{21u} = \frac{S_{21o}}{\sqrt{S^2_{21o}-2S_{21o}+2}} \text{Arg} \tan^{-1}(1-S_{21o})$$

$$S_{11u} = \frac{\sqrt{2}\, S_{11o}}{\sqrt{1+S^2_{11o}}} \text{Arg} \tan^{-1}\left(\frac{1-S_{11o}}{1+S_{11o}}\right)$$

$$S_{21L} = \frac{S_{21o}}{\sqrt{2}} \text{Arg } 45^{o}$$

$$S_{11L} = \sqrt{\frac{1+S^2_{11o}}{2}} \text{Arg} \tan^{-1}\left(\frac{1-S_{11o}}{1+S_{11o}}\right)$$

Figure 3 shows the above Qu and QL loci in the S_{11} and S_{21} plane of the SRXS configuration. S_{11} and S_{21} curves corresponding to $\beta = 1$ are also shown on the figure which can be directly used to determine Qu and QL of this configuration.

iii) Configuration SRXP (fig. 1c):

For this configuration

$$\beta = \frac{zo}{2R} = \frac{1-S_{21o}}{S_{21o}} = \frac{-S_{11o}}{1+S_{11o}}$$

and using the above approach the final relations to draw the Qu and QL loci on S_{11} and S_{21} plane are given by:

$$S_{11u} = \frac{S_{11o}}{\sqrt{S_{11o}^2+2S_{11o}+2}} \text{Arg} \tan^{-1}(1+S_{11o})$$

$$S_{21u} = \frac{\sqrt{2}\, S_{21o}}{\sqrt{1+S^2_{21o}}} \text{Arg} \tan^{-1}\left(\frac{1-S_{21o}}{1+S_{21o}}\right)$$

$$S_{11L} = \frac{S_{11o}}{\sqrt{2}} \text{Arg } 45^{o}$$

$$S_{21L} = \sqrt{\frac{1+S^2_{21o}}{2}} \text{Arg} \tan^{-1}\left(\frac{1-S_{21o}}{1+S_{21o}}\right)$$

Figure 4 shows the Qu and QL loci in the S_{11} and S_{21} plane of the SRXP configuration as well as the S_{11} and S_{21} curves corresponding to $\beta = 1$. Qu and QL can now be determined directly from either S_{11} or S_{21} plane.

iv) Configuration PRXS (fig. 1d):

In this case

$$\beta = \frac{R}{2Zo} = \frac{S_{11o}}{1-S_{11o}} = \frac{1-S_{21o}}{S_{21o}}$$

and the final relations to draw Qu and QL loci on the S_{11} and S_{21} plane are

$$S_{11u} = \frac{S_{11o}}{\sqrt{S_{11o}^2-2S_{11o}+2}} \text{Arg} \tan^{-1}(1-S_{11o})$$

$$S_{21u} = \frac{\sqrt{2}\, S_{21o}}{1+S^2_{21o}} \text{Arg} \tan^{-1}\left(\frac{1-S_{21o}}{1+S_{21o}}\right)$$

$$S_{11L} = \frac{S_{11o}}{\sqrt{2}} \text{Arg } 45^{o}$$

$$S_{21L} = \sqrt{\frac{1+S^2_{21o}}{2}} \text{Arg} \tan^{-1}\left(\frac{1-S_{21o}}{1+S_{21o}}\right)$$

Figure 5 shows the Qu and QL loci S_{11} and S_{21} plane of PRXS configuration as well as the S_{11} and S_{21} curves corresponding to $\beta =1$. Qu and QL can now be directly determined from the network analyzer polar display. The coupling coefficient can be determined from the measured S parameters using the corresponding relations given above for each configuration.

References

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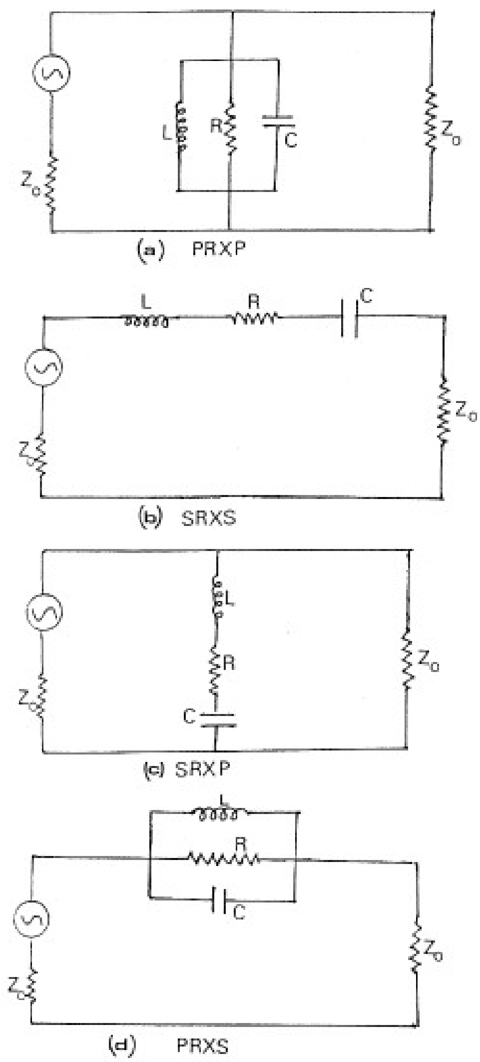

Fig. 1 The four configurations of a two port RLC resonant circuit

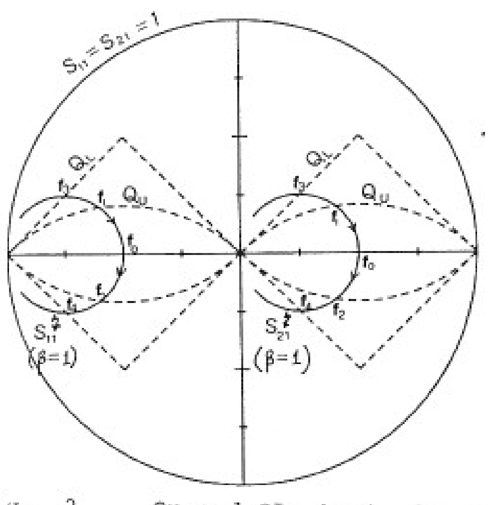

Fig. 2 Qu and QL loci for PRXP configuration

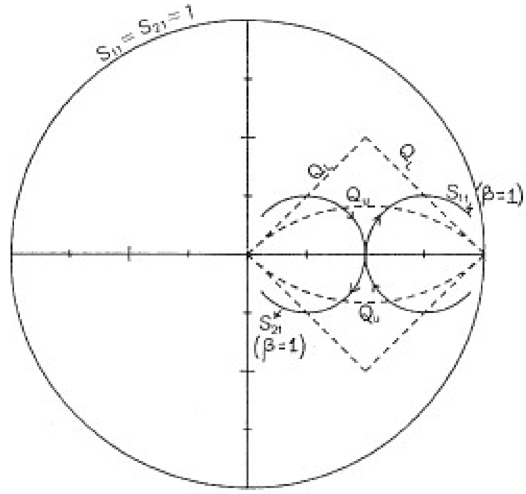

Fig. 3 Qu and QL loci for SRXS configuration

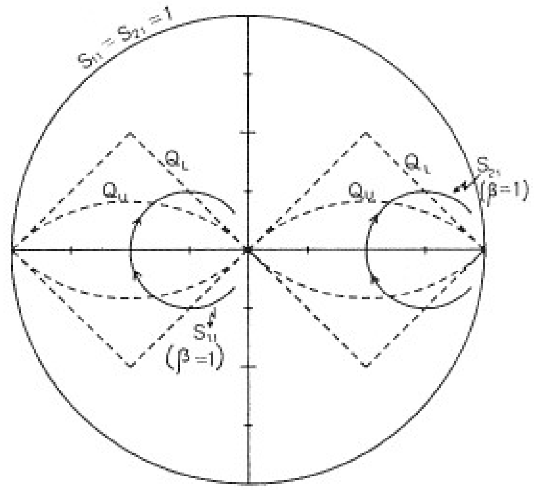

Fig. 4 Qu and QL loci for SRXP configuration

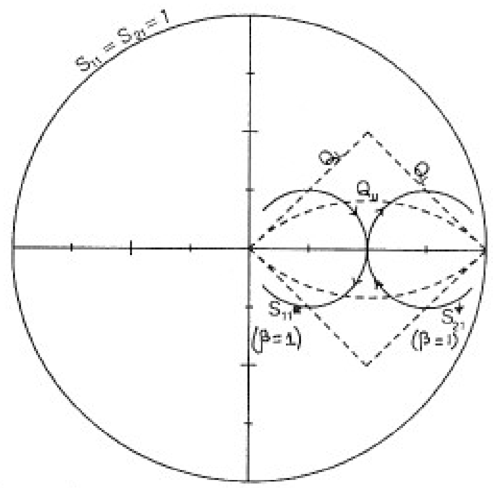

Fig. 5 Qu and QL loci for PRXS configuration